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LETTER TO THE EDITOR

Critical dynamics of Heisenberg spins on self-avoiding-walk chains

S Bhattacharya and B K Chakrabarti

Saha Institute of Nuclear Physics, 92 Acharya Prafulla Chandra Road, Calcutta–700 009, India

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Abstract. The dynamical exponent z for the critical spin-wave dynamics of nearest-neighbour interacting Heisenberg spins on a self-avoiding-walk (SAW) chain is estimated here using a scaling picture and also applying a real-space renormalisation group technique to some quasilinear fractal lattices. The results indicate $z=2Dt$, where D is the fractal dimensionality of the SAW chain lattice and t is the exponent for the length of the shortest nearest-neighbour connecting path of the SAW.

Recently we have proposed and studied the static critical properties of an Ising model with nearest-neighbour interactions on self-avoiding-walk (SAW) chains, where the SAWs themselves are executed on any d-dimensional lattice (Chakrabarti and Bhattacharya 1983, Bhattacharya and Chakrabarti 1984). An application of this important lattice statistical model may be in the study of phase transitions of magnetic polymers where each monomer of a linear polymer possesses localised magnetic moments (the monomer–monomer repulsion should be strong enough to resist the collapse of the SAW structure induced by the interactions between neighbouring monomers not connected by chemical bonds). Study of this model should also be interesting in connection with studies on magnetism of disordered solids near the percolation threshold, since a typical SAW configuration with nearest-neighbour connections can be considered equivalent to a 'generator' (links and blobs) of the backbone of the infinite percolating cluster (see, for example, Bhattacharya and Chakrabarti 1984). Our studies, mentioned above, indicated a finite transition temperature for such nearest-neighbour interacting Ising models on SAW chains and also a new static critical behaviour different from that of a one-dimensional Ising chain.

In this letter we investigate the critical dynamics of nearest-neighbour interacting Heisenberg spins on SAW chains at zero temperature. Our analysis indicates that the dynamic exponent $z=2Dt$, where D is the fractal dimensionality of the SAW lattice (Mandelbrot 1982) and t is the exponent for the shortest connection length of a SAW (Bhattacharya and Chakrabarti 1984).

Let us consider a sufficiently large section (so that scaling laws hold good) of N steps of the SAW, as shown in figure 1. The Heisenberg spins on it are interacting with nearest-neighbour interactions (J), so that the spin dynamics primarily follow the shortest nearest-neighbour connecting path from one end to the other of the SAW. Such a path is also indicated in figure 1, and has a length $L_N(<N) \sim N^t$ (Bhattacharya and Chakrabarti 1984). Since along such a path, the dynamics is essentially like that

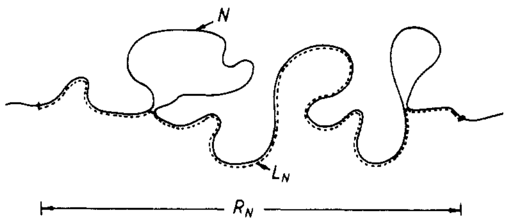

Figure 1. A typical SAW segment (full curve) of size N showing the shortest nearest-neighbour connecting path (broken curve), of length $L_N \sim N^t$, through which the dynamics will primarily propagate. The average end-to-end distance is $R_N \sim N^\nu$.

of a linear chain, the low-energy magnons will have a dispersion

$$\omega(Q) = \tfrac{1}{2}JQ^2, \tag{1}$$

where the magnon wavevector Q is defined along the dynamical path of length L_N which scales, compared to the (Euclidean) end-to-end distance $R_N(\sim N^\nu)$ as $R_N^{t/\nu}$ for $Q^{-1} < R_N$, and as $N^{(\nu-t)}Q$ for $Q^{-1} \geqslant R_N$; ν being the average end-to-end distance exponent for the SAW. The reduced wavevector q (defined in the Euclidean space) is, therefore, related to the dynamical wavevector Q as (cf Ziman 1979)

$$q = \begin{matrix} N^{(t-\nu)}Q, & \text{for } q \leqslant q_c \sim N^{-\nu} \\ Q^{\nu/t}, & \text{for } q > q_c, \end{matrix} \tag{2}$$

giving the hydrodynamic form for the dispersion as

$$\begin{matrix} \omega(q) = q^2 f(q\xi), & \xi \sim R_N \\ f(y) \sim y^{z-2}, & y \to \infty, \end{matrix} \tag{3}$$

where the dynamical exponent $z = 2t/\nu$. Expressing ν^{-1} as the fractal dimensionality D of the SAW lattice (Mandelbrot 1982), the dynamical exponent z may be expressed as

$$z = 2Dt \tag{4}$$

which, for example, for SAWs on two-dimensional lattices takes the value 2.61, since $\nu = D^{-1} = 0.75$ (Nienhuis 1982) and $t \approx 0.977$ (Bhattacharya and Chakrabarti 1984).

One can also justify the above expression for the dynamical exponent z, applying real-space renormalisation group (RSRG) technique to study critical dynamics (Stinchcombe 1983) of quasi-linear self-similar lattices like those shown in figure 2 (see, for example, Gefen *et al* 1983). Heisenberg spins are placed on the sites along the length of such lattices and are assumed to interact with their nearest neighbours of the same segment (or the 'generator') with exchange J. The equation of motion for the transverse spin components S_i, at site i is:

$$\Omega S_i = \sum_j K_{ij}(S_i - S_j), \tag{5}$$

where $\Omega = \omega/J$, ω is the characteristic frequency and $K_{ij} = 1$ if i, j are nearest neighbours and zero otherwise.

We now apply the RSRG technique of Stinchcombe to the lattice in figure 2(a), with a scale factor $b = 3$. The corresponding renormalised dynamic variable Ω' is formulated in such a way that the dynamic relationships between the remaining

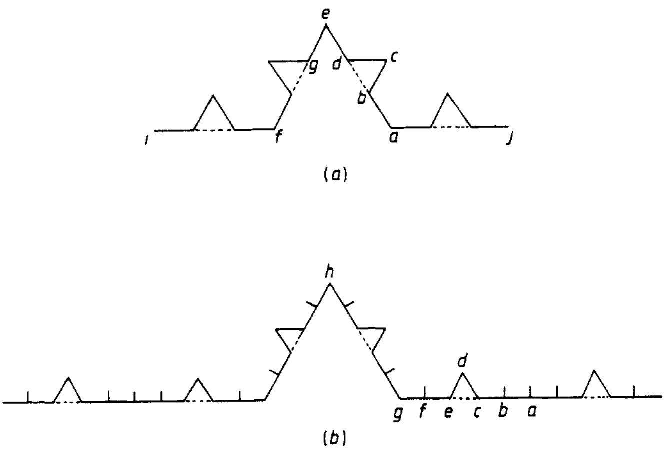

Figure 2. Quasilinear fractal lattices formed by repeating the basic unit or the 'generator' ('*ae*' in (*a*) and '*ag*' in (*b*)) in a self-similar way. The additional nearest-neighbour interactions are shown by broken lines. The fractal dimension D for the lattice (*a*) is $\ln 4/\ln 3$ and that for lattice (*b*) is $\ln 6/\ln 5$.

variables (between spins $\langle aj\rangle$, $\langle ae\rangle$, $\langle ef\rangle$, $\langle fi\rangle$ and also $\langle af\rangle$) are preserved. Following the same intermediate transformations as that used in Gefen *et al* (1983) for the thermal problem of Ising spins on the lattice of figure 2(*a*), we obtain

$$\Omega' = 16\Omega - 20\Omega^2 + 8\Omega^3 - \Omega^4. \tag{6}$$

Linearising this recursion relation (6) around the fixed points value Ω^* $(=0)$ of Ω one obtains the dynamic exponent z as:

$$z = 2(\ln 4/\ln 3) = 2D, \tag{7}$$

where D is the fractal dimensionality of the lattice of figure 2(*a*). Employing the same RSRG method on the fractal lattice of figure 2(*b*) one obtains exactly the same result i.e. $z = 2D$ ($D = \ln 6/\ln 5$ for the lattice in figure 2(b)). Such an expression compares exactly with the expression (4), if $t = 1$ for such quasilinear fractal lattices. In fact, for such lattices the length of the shortest nearest-neighbour connecting path is just a finite fraction of the total length of the lattice, giving $t = 1$ exactly in such cases.

References

Bhattacharya S and Chakrabarti B K 1984 *Z. Phys.* B (to be published)
Chakrabarti B K and Bhattacharya S 1983 *J. Phys. C: Solid State Phys.* **16** L1025
Gefen Y, Aharony A and Mandelbrot B B 1983 *J. Phys. A: Math. Gen.* **16** 1267
Mandelbrot B B 1982 *The Fractal Geometry of Nature* (San Francisco: Freeman) p 329
Nienhuis B 1982 *Phys. Rev. Lett.* **49** 1062
Stinchcombe R B 1983 *Phys. Rev. Lett.* **50** 200
Ziman T A L 1979 *J. Phys. C: Solid State Phys.* **12** 2545